Tecniche di controllo robusto l_1 e l_∞ per la regolazione del minimo nei motori a scoppio

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Outline

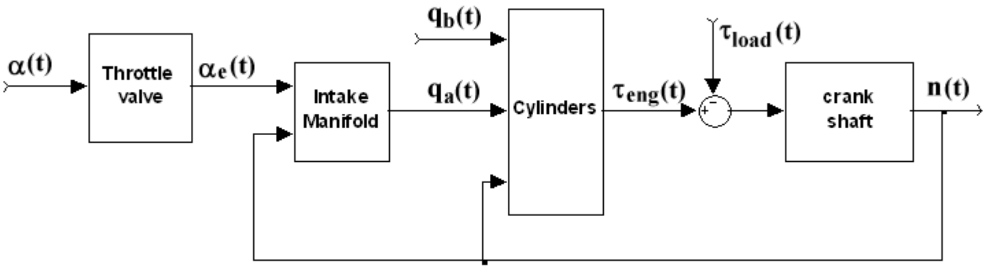
- Idle speed control problem
- Port-Injection engines description
- Proposed linear control structure
- l_∞ and l_1 control via the polynomial equation approach
- Experimentation
- Conclusions

Idle speed control

- It is in charge of handling all situations in which the gas pedal is released. In particular:
  - Low regimes during driving, no matter the transmission gear engaged;
  - High loads with first gear engaged, e.g. vehicles almost still or slowing moving in steep slops;
  - Engine speed fast dropping from high rpm to the one prescribed at the idle;
  - Idle gear and variable loads acting on the crankshaft;

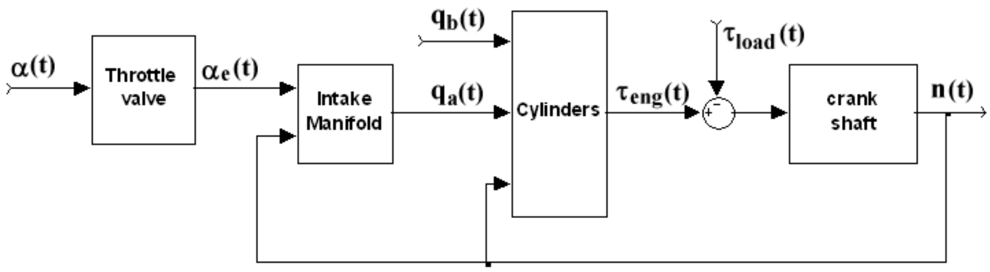
- In all above situations the control problem consists of
  - preventing engine stalls
  - maintaining the engine speed at the prescribed rpm;
  - the rejection of load disturbances;

A port-injection gasoline engine model



- Four interacting subsystems are of interest:
  - the **throttle valve**
  - the **intake manifold**
  - the **cylinder**
  - the **crankshaft**

The throttle valve dynamics



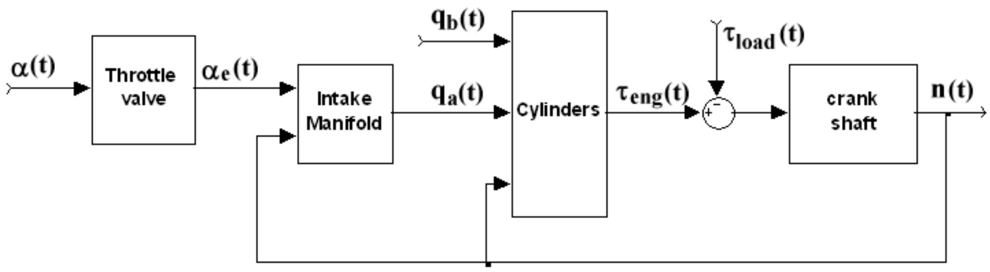
The dynamic of the *throttle valve* is modelled by a first-order lag with input delay:

$$\dot{\alpha}_e(t) = \frac{1}{\tau_\alpha}\alpha_e(t) + \frac{1}{\tau_\alpha}\alpha(t - d_\alpha)$$

where:

- α denotes the throttle valve command (gas pedal)
- α_e denotes the throttle valve angle
- $d_\alpha = 20\ ms$ denotes the electrical actuator delay
- $\tau_\alpha = 50\ ms$ mechanic time constant

The intake manifold dynamics



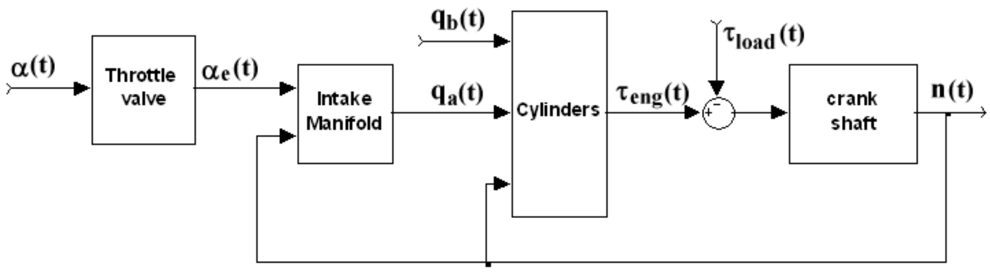
The *intake manifold* dynamic is described in terms of the manifold pressure p and of the amount of air in the cylinder q_a as follows:

$$\begin{aligned}\dot{p}(t) &= K_{gas}(F_{th}(\alpha_e(t), p(t)) - F_{cyl}(n(t), p(t))) \\ \dot{q}_a(t) &= F_{cyl}(n(t), p(t))\end{aligned}$$

where:

- $F_{th}(\alpha_e, p)$ is the input air-flow rate. It is a highly nonlinear static function, approximated by a piece-wise linear function of α_e and p.
- $F_{cyl}(p, n)$ is the output air-flow rate. It is a highly nonlinear static function, approximated by a piece-wise linear function of p and n.
- K_{gas} is the gas constant

The cylinder dynamics



The cylinder subsystem describes how the torque is generated from fuel combustion. A static map of the form

$$\mathcal{T}_{eng} = \mathcal{T}_{eng}(q_a, q_b, n, \beta)$$

is usually achieved experimentally where

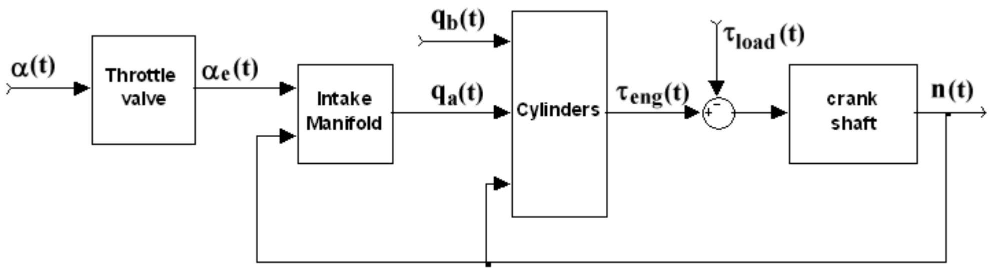
- q_a and q_b are the total masses of injected fuel and air;
- n the engine speed and β the spark advance;

A more convenient way to express the above map at the stoichiometric ratio $\lambda = \frac{q_a}{q_b} \approx 14.66$ (for gasoline) is

$$\mathcal{T}_{eng} = \mathcal{T}_{pot}(q_a, n)\eta(\beta)$$

where $\mathcal{T}_{pot}$ is the maximum potential torque and $\eta(\beta)$ the spark advance efficiency

The crankshaft dynamics



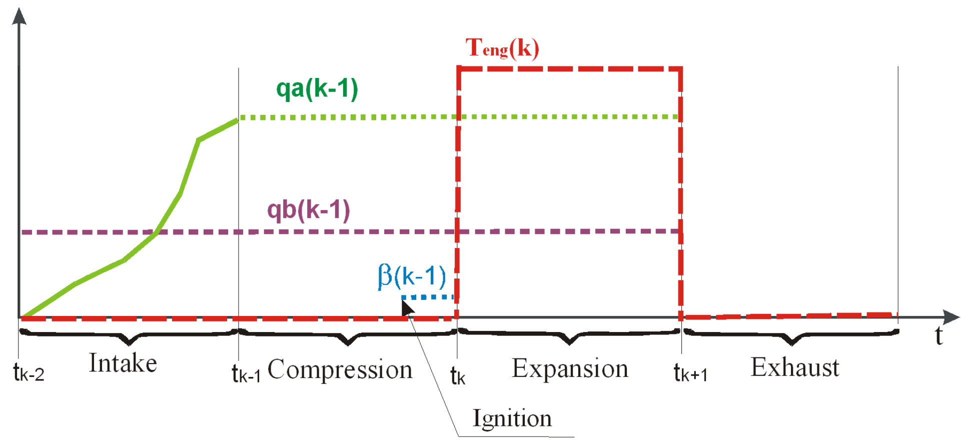
The crankshaft block describes the evolution of the crankshaft revolution speed n, whose acceleration depends on the difference between the engine torque $\mathcal{T}_{eng}$ and the load torque $\mathcal{T}_{load}$:

$$\dot{n}(t) = K_J(\mathcal{T}_{eng}(t) - \mathcal{T}_{load}(t))$$

The load torque $\mathcal{T}_{load}(t)$ consists essentially of three distinct amounts:

- Pumping torque
- Friction torque
- Additional torque, due to the auxiliary subsystems powered by the engine (e.g. electrical generator, air conditioner, etc.)

Spark ignition engine cycle



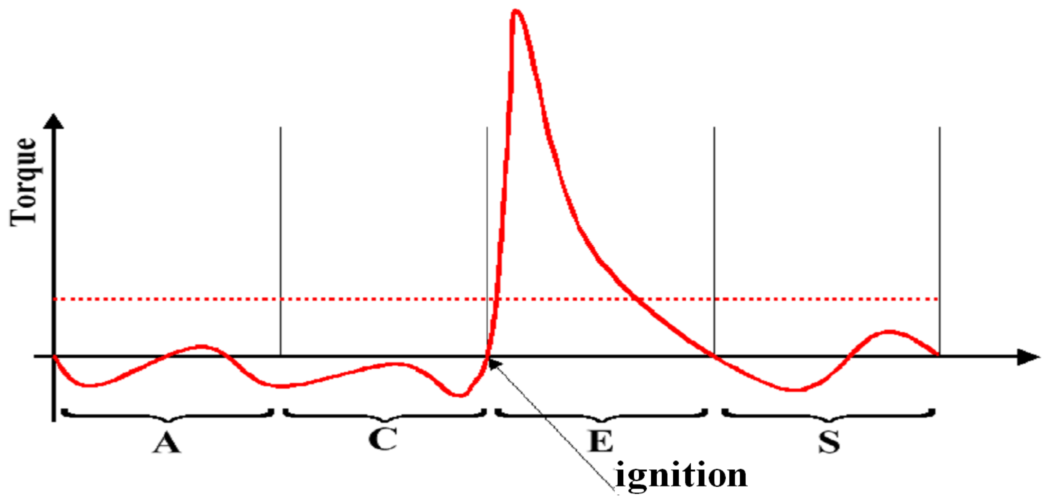
- The dead center events of a four-stroke engine[1] occur when the pistons reach either the top or bottom positions. We denote by t_k the sequence of times at which they occur.
- Then, the amount of air q_a loaded by a cylinder during each intake stroke is obtained by integrating the input air-flow F_{cyl} between two dead centers, i.e.

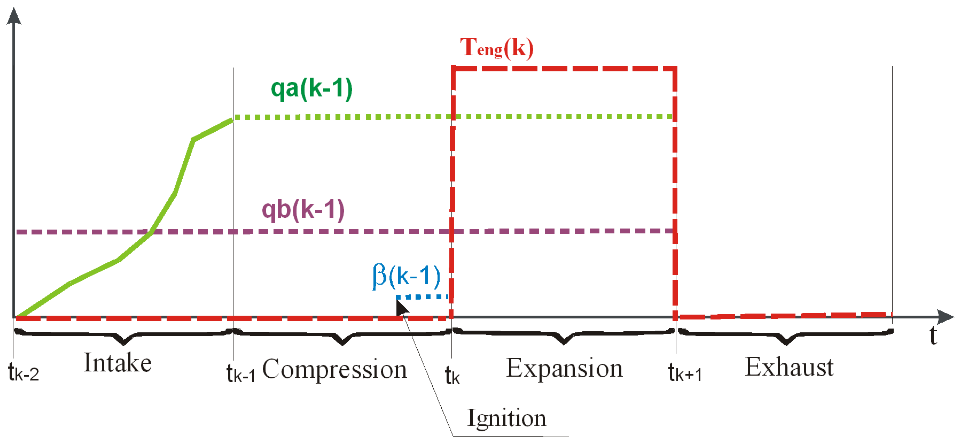
$$q_a(t_{k-1}) = \int_{t_{k-2}}^{t_{k-1}} F_{cyl}(n(t), p(t))dt$$

- We assume that $q_a(t) = q_a(t_{k-1}),\ \forall t \in [t_{k-1}, t_{k+1}]$ is constant during subsequent compression and expansion strokes

[1] Intake, compression, expansion and exhaust strokes.

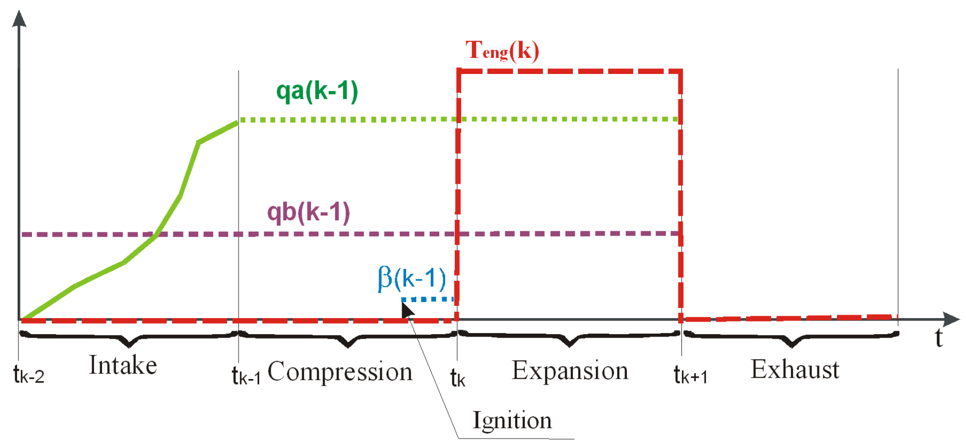
An averaged modelling approach





The torque generated during the expansion stroke is averaged along expansion stokes

The hybrid torque generation model



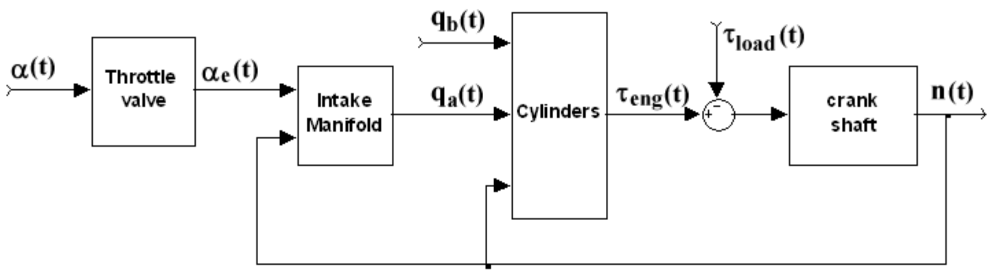
- In the hybrid model the produced torque $\mathcal{T}_{eng}(t)$ is modelled as a piecewise-constant signal, synchronized with the dead center events.

  $$\mathcal{T}_{eng}(t) = \mathcal{T}_{eng}(t_k) = \mathcal{T}_{pot}(q_a(t_{k-1}), n(t_k))\eta(\beta(t_{k-1})), \ \ t \in [t_k, t_{k+1})$$

  where:

  - $q_a(t_{k-1})$ is the total mass of injected air at the end of the intake stroke. Of course $q_b(t_{k-1}) = \lambda q_a(t_{k-1})$;
  - $n(t_k)$ is the value of the engine speed at the beginning of the stroke t_k;
  - $\beta(t_{k-1})$ is the spark advance for the expansion stroke t_k decided at time t_{k-1}

Discrete-time multirate system



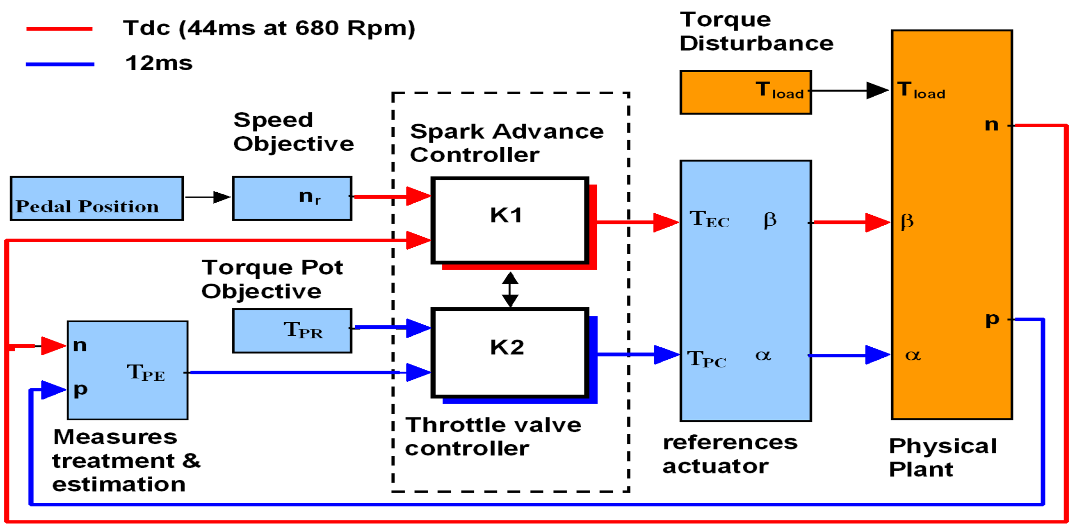
- The throttle valve and intake manifold dynamics are discretized at the fast and constant sampling rate $t_f = 12$ ms. The throttle valve commands are provided at even faster sampling rates (4 ms).

- All other dynamics are discretized at every engine stroke (in four-cylinders engines) at variable TDC sampling rates. This correspond to the sampling rate of $t_k = 44$ ms at the speed of 680 rpm. The spark advance commands are also provided at TDC sampling rates.

- A multirate discrete-time LTI plant description is enough for control synthesis purposes because mostly of the nonlinearities can be inverted. The TDC discretized system describes all relevant quantities at dead-center times.

- The model is built up at the nominal idle speed of 680 rpm. Variability in t_k are taken into account but this is not a serious problem for the idle speed control

Discrete-time multirate control structure



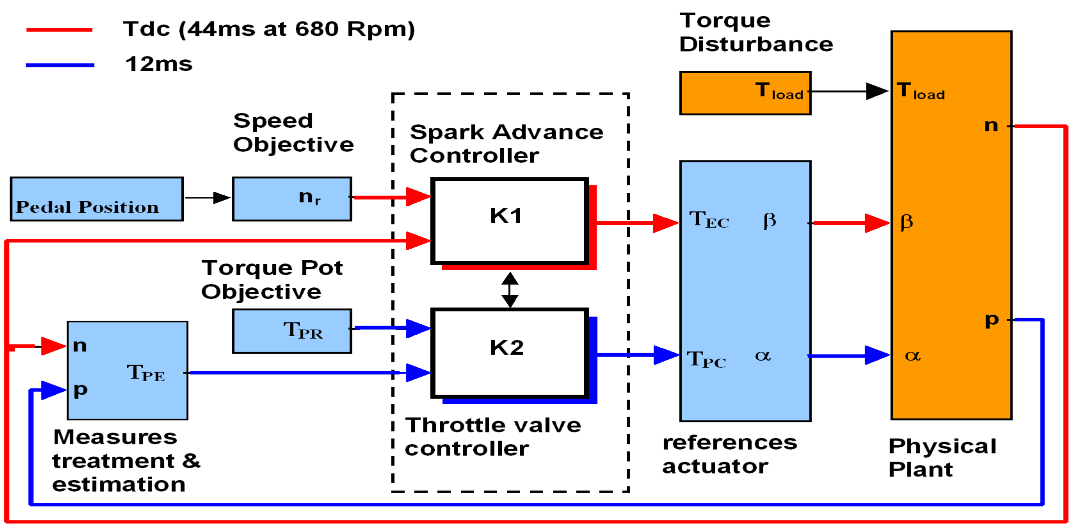
- The **Spark Advance** and **Throttle Valve** SISO controllers have be synthesized on the basis of the following multirate LTI-TD plant description

$$
\begin{aligned}
n(t_k) &= \frac{B_1(d)}{A_1(d)} T_{ec}(t_k) + \frac{C_1(d)}{A_1(d)} T_{load}(t_k), \quad T_{ec} \leq T_{pc} \\
T_{pe}(t_f) &= \frac{B_2(d)}{A_2(d)} T_{pc}(t_f)
\end{aligned}
$$

  where

  - T_{ec} is the required produced torque;
  - T_{pc} is the required potential torque;
  - T_{load} is the total load torque;
  - T_{pe} is an estimate of the actual potential torque;

Discrete-time multirate control structure

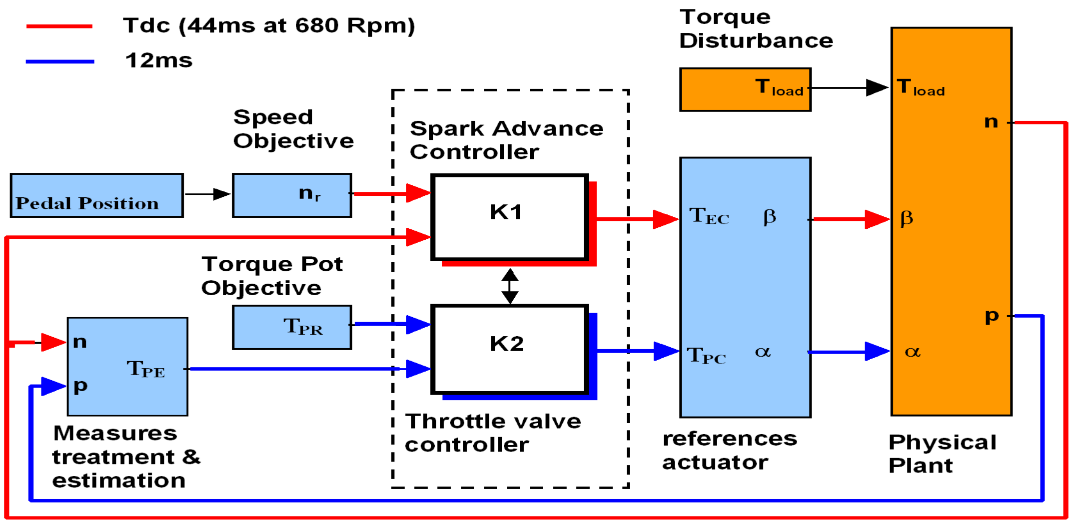


- The **SA** controller is in charge to regulate the engine speed. Its main goal is fast rejection of step disturbances T_{load}. To reduce consumption, low activity to the command T_{ec} is required.

- The **TV** controller is in charge to regulate the dynamic of the required potential torque T_{pr}, to be considered as an instantaneous torque reserve for fast compensation of load disturbance T_{load}. Its main goal is to provide a good tracking of T_{pr} by T_{pe}.

- The **reference actuator** block is in charge to translate the T_{ec} and T_{pc} requirements in terms of spark advance β and throttle valve angle α. Moreover, all nonlinearities are here inverted.

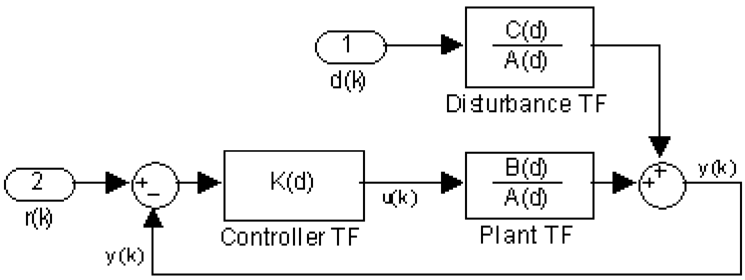
Spark advance controller design



- Fast rejection of piecewise constant load disturbances
- Fuel consumption minimization during transients
- Good tracking performance on the engine speed
- Industrial practice typically makes use of PID-like or other no model based control design techniques

- l_∞ and l_1 finite-dimensional optimal control

A polynomial equation approach



Assuming for simplicity $r(k) = 0$

$$Y(d) = \frac{B(d)}{A(d)}U(d) + \frac{C(d)}{A(d)}D(d)$$

- d is the one-step delay,
- $U(d)$, $Y(d)$ and $D(d)$ $\mathcal{D}$-transforms of input, output and disturbance,
- $\frac{B(d)}{A(d)}$ strictly causal and $\frac{C(d)}{A(d)}$ causal

Assume that the disturbance sequence $d(t)$ is a polynomially unbounded sequence with rational $\mathcal{D}$-transform

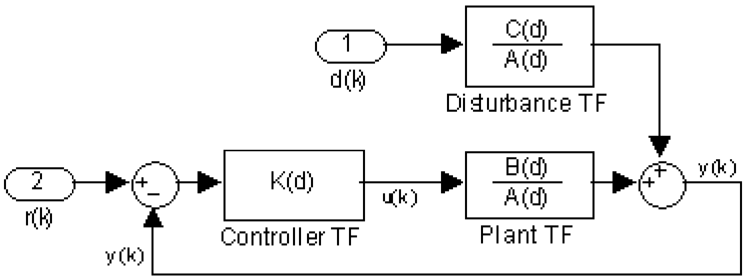
$$D(d) := \frac{B_d(d)}{A_d(d)}$$

with roots of $A_d(d)$ in $|d| \geq 1$.

Assume also:

$$(\mathbf{A.1}) \begin{cases} (A, B) \text{ coprime with } A(0) \neq 0,\ B(0) = 0 \\ (A_d, B_d) \text{ coprime with } A_d(0) \neq 0. \end{cases}$$

A polynomial equation approach



Define the feedback action between the output $y(t)$ and $u(t)$ as $U(d) = -\mathcal{K}(d)Y(d)$ with

$$\mathcal{K}(d) = \frac{S(d) + A(d)\mathcal{Q}(d)}{R(d) - B(d)\mathcal{Q}(d)}$$

with the polynomial pair (R, S) satisfying

$$A(d)R(d) + B(d)S(d) = 1$$

and the free Youla transfer function $\mathcal{Q}$ causal and asymptotically stable.

Perform the following causal/anticausal decompositions

$$B = B^-B^+, \quad A_d = A_d^-A_d^+, \quad B_d = B_d^-B_d^+, \quad C = C^-C^+$$

where

- B^+ is stable, viz. free of roots in $|d| \leq 1$) and
- B^- is monic unstable, viz. with all of its roots in $|d| \leq 1$

Deadbeat ripple-free parameterization

Assume

$$(\mathbf{A.2}) \begin{cases} (A_d, B) \text{ coprime polynomial pair} \\ A_d \text{ factor of } (1-d)C^-, \text{ i.e.} \\ (1-d)C^- = GA_d, \text{ for some polynomial } G. \end{cases}$$

The first assumption is required to ensure both the dead-beat and ripple-free properties, whereas the second needs only if ripple-free responses are of interest.

Proposition - Let (A.1)-(A.2) be fulfilled. Then, the Youla parameter $\mathcal{Q}$ yielding all ripple-free dead-beat controllers and the corresponding closed-loop responses $Y(d)$ and $\Delta U(d) = (1-d)U(d)$ can be parameterized in terms of an arbitrary polynomial $W(d)$ as follows

$$\mathcal{Q} = \frac{Z_o + A_d\,(T_o + B^+W)}{C^+B^+B_d^+} \tag{1}$$

$$Y = Y^o - C^-B^-B_d^-[T_o + B^+W] \tag{2}$$

$$\Delta U = GB_d^-\left(SC^+B_d^+ + A\,[V_o + A_dW]\right) \tag{3}$$

$$\text{with } G \text{ as in (A.2)}$$

where (Y_o, Z_o) is the unique m.d. solution w.r.t. with Y (i.e. $\deg Y < \deg C^-B^-B_d^-$) of

$$ZC^-B^-B_d^- - A_dY = CB_dR$$

while (V_o, T_o) is the unique m.d. solution w.r.t. T (i.e. $\deg T_o < \deg B^+$) of

$$-A_dT + B^+V = Z_o$$

Design objectives - Performance

Observe that the degrees of both $Y(d)$ and $\Delta U(d)$ grow ip monotonically with the degree w of $W(d)$. Thus, w is a control design parameter

<u>Fast disturbance rejection</u>

(P.1) $\min_{W\in\Re^w[d]} \|Y\|_{\mathcal{A}_\infty}$

(P.2) $\min_{W\in\Re^w[d]} \|Y\|_{\mathcal{A}_\infty}$ subject to $\|\Delta U\|_{\mathcal{A}_\infty} < \gamma_1$

<u>Minimization of the control effort</u>

(P.3) $\min_{W\in\Re^w[d]} \|\Delta U\|_{\mathcal{A}_\infty}$

(P.4) $\min_{W\in\Re^w[d]} \|\Delta U\|_{\mathcal{A}_\infty}$ subject to $\|Y\|_{\mathcal{A}_\infty} < \gamma_2$

- $\|H(d)\|_{\mathcal{A}_\infty} := \|h_k\|_\infty$ where $H(d) := \sum_{k=0}^{\infty} h_k d^k$
- For all problems the cost monotonically decreases as $w \to \infty$
- All formulations give rise to finite dimensional linear programming problems

Design objectives - Robustness

Under additive unstructured causal LTV perturbations $\Delta\mathcal{P}_\gamma$, with $\|\Delta\mathcal{P}_\gamma\|_\mathcal{A} < \gamma$ one has that

$$\frac{\|Y_\gamma - Y\|_{\mathcal{A}_\infty}}{\|Y\|_{\mathcal{A}_\infty}} \leq \frac{\gamma\|\mathcal{M}\|_\mathcal{A}}{1-\gamma\|\mathcal{M}\|_\mathcal{A}}$$
$$\frac{\|U_\gamma - U\|_{\mathcal{A}_\infty}}{\|U\|_{\mathcal{A}_\infty}} \leq \frac{\gamma\|\mathcal{M}\|_\mathcal{A}}{1-\gamma\|\mathcal{M}\|_\mathcal{A}}$$

where $\mathcal{M}$ is the nominal control sensitivity function and $\|H(d)\|_\mathcal{A} = \|h_k\|_1$ with $H(d) := \sum_{k=0}^{\infty} h_k d^k$

Then, the upper-bound on the maximum relative errors can be made as small as possible by minimizing $\|\mathcal{M}\|_\mathcal{A}$. In fact, $\gamma\|\mathcal{M}\|_\mathcal{A} \ll 1$ implies

$$\frac{\gamma\|\mathcal{M}\|_\mathcal{A}}{1-\gamma\|\mathcal{M}\|_\mathcal{A}} \approx \gamma\|\mathcal{M}\|_\mathcal{A}$$

The nominal control sensitivity function

$$\mathcal{M} = \frac{U(d)}{B_d/A_d} = \frac{M_1(d) + M_2(d)W(d)}{B_d^+}$$

is a polynomial too provided that either

- B_d^+ is a factor of both polynomials M_1 and M_2
- B_d^+ is a scalar

Design objectives - Robustness

Robust designs

(P.5) $\min_{W \in \Re^w[d]} \|\mathcal{M}\|_{\mathcal{A}}$

(P.6) $\min_{W \in \Re^w[d]} \|Y\|_{\mathcal{A}_\infty}$ subject to $\|\mathcal{M}\|_{\mathcal{A}} < \gamma_3$

Robust minimization of the control effort

(P.7) $\min_{W \in \Re^w[d]} \|\Delta U\|_{\mathcal{A}_\infty}$ subject to $\|\mathcal{M}\|_{\mathcal{A}} < \gamma_4$

(P.8) $\min_{W \in \Re^w[d]} \|\mathcal{M}\|_{\mathcal{A}}$ subject to $\|\Delta U\|_{\mathcal{A}_\infty} < \gamma_5$

- For all problems the cost monotonically decreases as w goes to ∞
- All formulations give rise to finite dimensional linear programming problems

Experimental results

- The control structure has been implemented on the ECU of a commercial 1.4L Volkswagen Polo engine

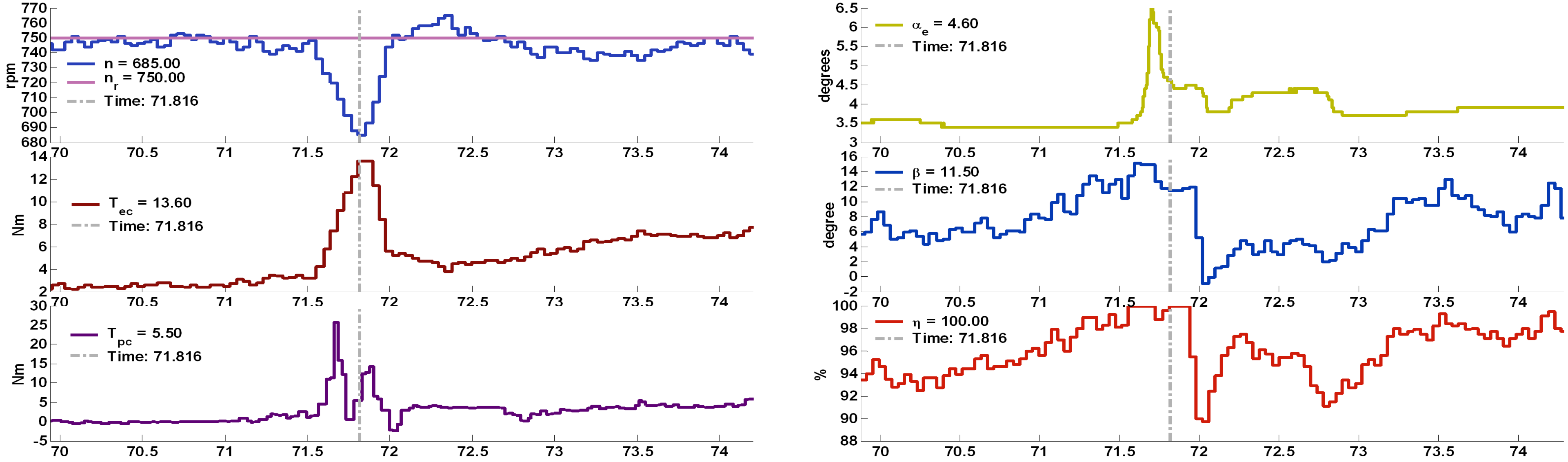
- The spark advance controller has been synthesized by minimizing the control effort
$$\min_{W \in \Re^w[d]} \|\Delta U\|_{\mathcal{A}_\infty}$$

- Step load disturbances have been considered, viz.
$$\frac{B_d}{A_d} = \frac{1}{(1-d)}$$

- The orders of the SA and TV controllers were 5 and, respectively, 3

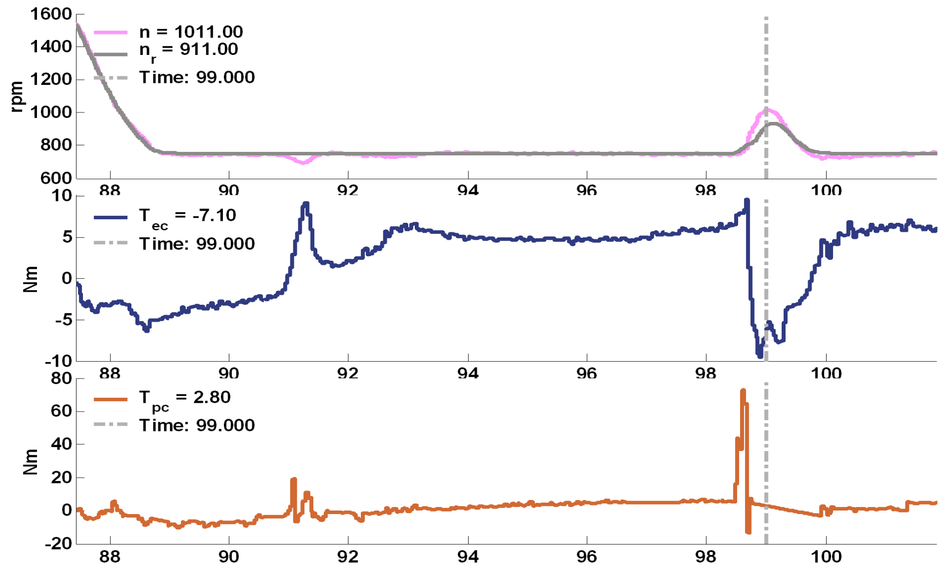
- Several tests have been accomplished
  - Response to load step disturbances
  - Transients towards idle
  - Rapid variations of the reference speed

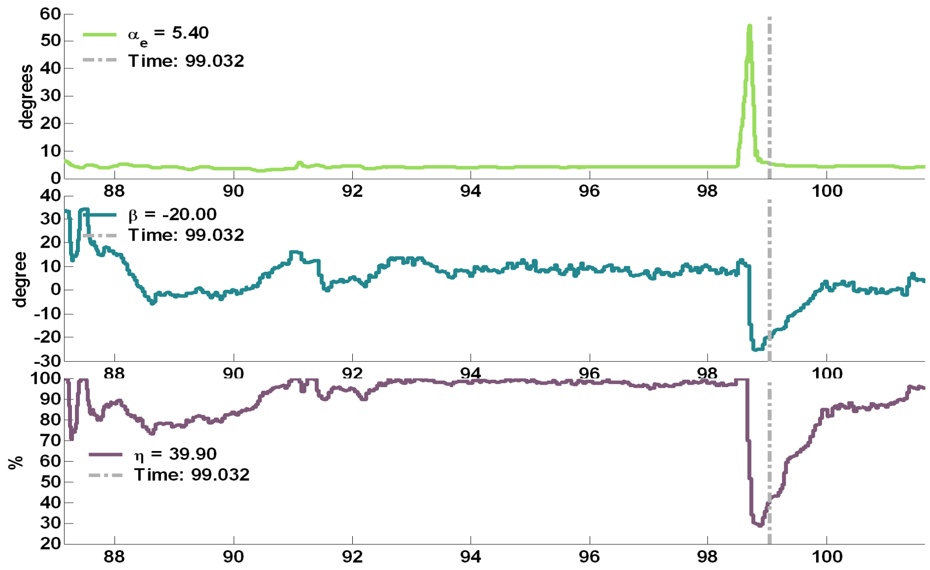
Response to load step disturbances



- Step disturbance ad time $t = 71.5$
- Overshoot on the engine speed halved w.r.t. PID/LQ control
- Believed mainly due to the penalization of $\|\Delta U\|_{\mathcal{A}_\infty}$
- Modest fluctuation of the idle speed around the target value
- No saturation on the spark advance efficiency
- Other commands rather smooth

Transients towards idle

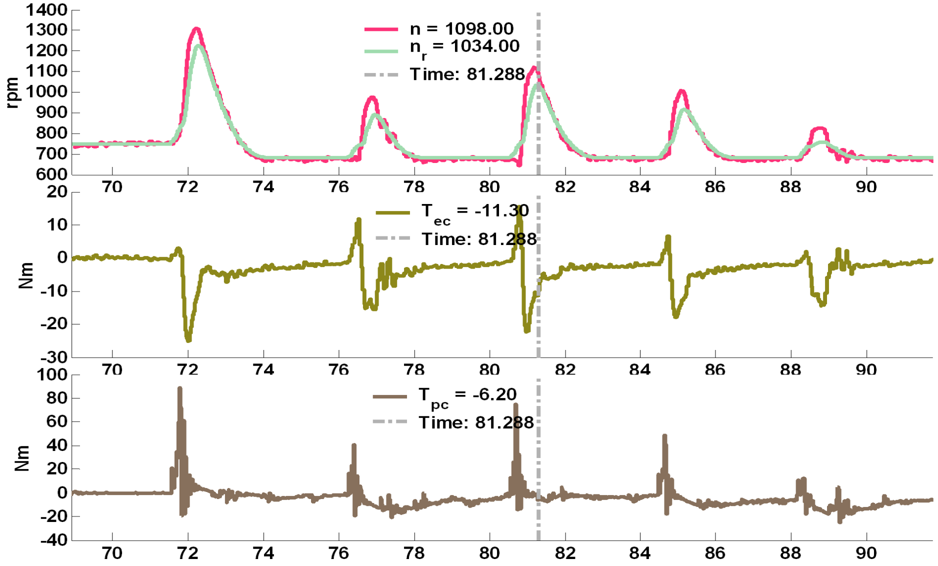


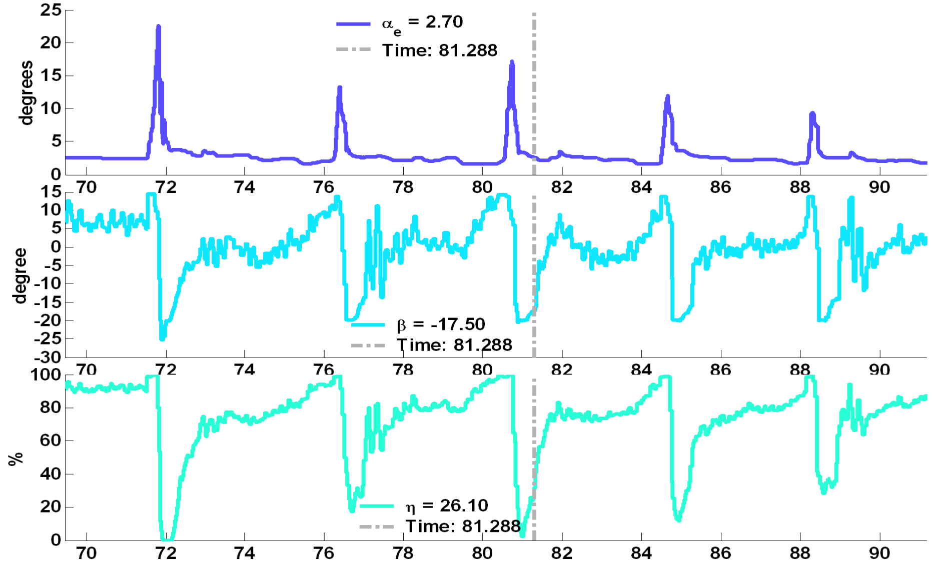


- The engine speed drops from high rpm to the idle speed reference value
- Transient is fast, smooth and well dumped
- In this test PID/LQ controllers usually exhibit large undershoots

- Also the commands are reasonable
- A gas pedal stroke at the end the plot causes a fast change of the reference speed
- A fast and smooth transient follows

Fast changes of the reference speed





- Usually a severe test
- Fast reference speed changes caused by gas pedal strokes
- PID/LQ usually produces large undershoots and fluctuations around the nominal idle speed
- Here, on the contrary, the responses are bloody good
- Neither undershoots nor fluctuations are practically observed
- Also the control effort is small

Conclusions

- l_∞ and l_1 optimal control techniques have been shown of potential interest for idle speed control problems in automotive industry

- Design techniques based on the polynomial equation approach make this class of controllers easily understandable and solvable with standard mathematical tools

- Also it allows to have some free control design parameters for both modulating the numerical burdens and permitting a fine tuning of the controllers in road tests

- Remarkable improvements w.r.t. to PID/LQ control have been reported by Magneti Marelli Powertrain's experts

- The control structure has been patented by Magneti Marelli Powertrain and it is actually used in some of its commercial ECUs